

Σ Business Calculus with Excel

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2.2 Modeling Revenue, Costs, and Profit

[Link to worksheets used in this section](#)

In the last section we looked at the economic model for supply and demand. We were particularly interested in the point of market equilibrium. In this section we will look at the model for revenue, cost and profit. As with the previous section we will begin with assumptions that make as many things as possible linear.

Revenue and a review of demand price:

The simple model for revenue is

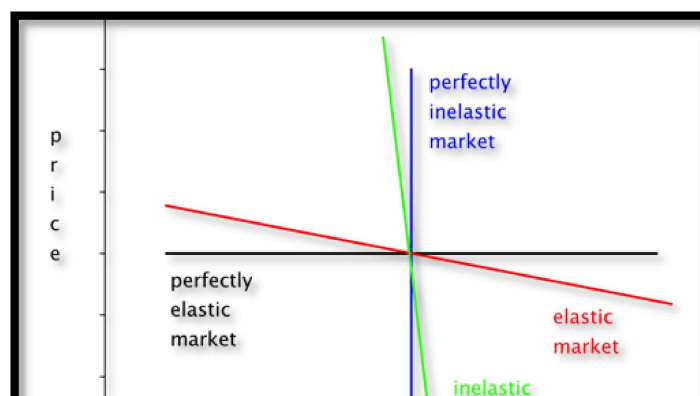
$$\text{revenue} = \text{quantity} * \text{price}.$$

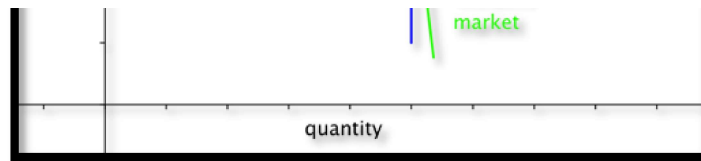
However, in the previous section we worked with two price functions, the supply price and the demand price. Since we can only make a sale if the consumer is willing to buy, we typically use the demand price in computing revenue. Our model is now

$$\text{revenue} = \text{quantity} * \text{demand price}(\text{quantity}).$$

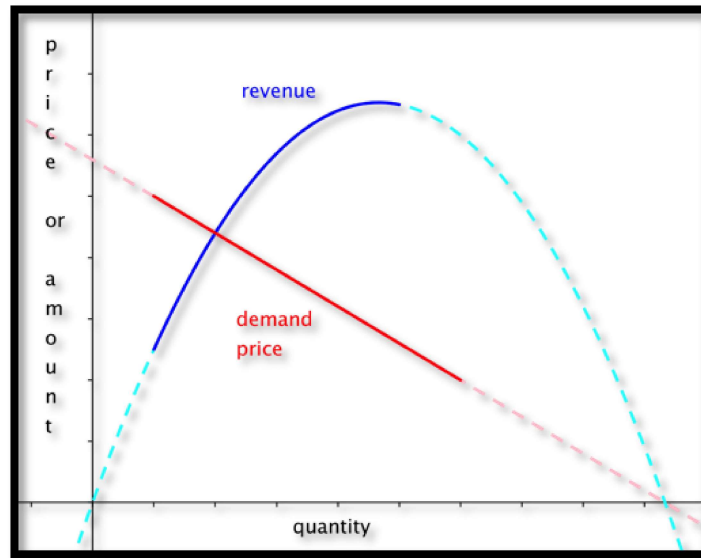
If the demand price is a linear function, then revenue is a quadratic function.

We previously noted that a linear demand price function has a negative slope. We should note the two limiting cases. If the slope of the demand curve is 0, the consumers have a fixed price they will pay for however much of the product is available. In this case the demand curve is a constant, so the revenue curve will be linear. This is referred to as a perfectly elastic market. The other limiting case is where the demand is for a fixed amount no matter what the price. In this case the demand curve is a vertical line and is not a function, so the revenue curve also fails to be a function of quantity.





- Obviously, we don't expect to find the limiting cases in the real world. In real world cases the revenue function has a negative coefficient for the quadratic term and is a downward facing parabola.

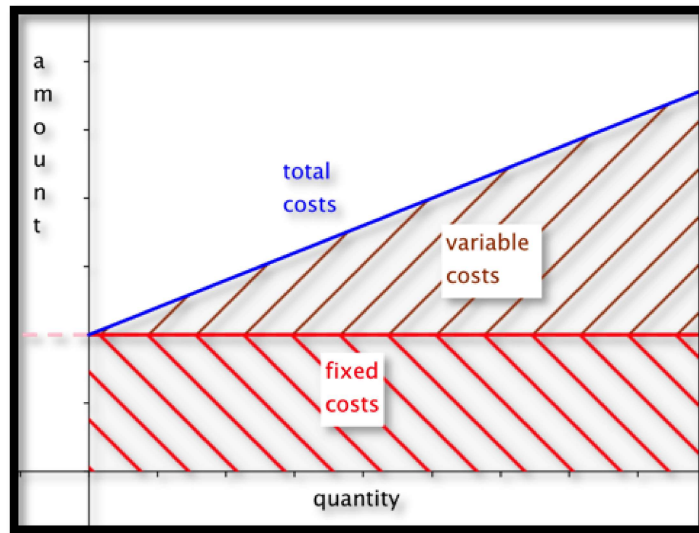


Example 2.2.1. Finding Revenue From Linear Demand Price.

Cost:

- Once again we will start with a simplified model for cost.

- For our (simplified) model we will break costs into **fixed costs** and **variable costs**.
- Fixed costs include the costs of being in business. They might include license fees, rent for a store or plant, and the cost of furnishings and equipment.
- Variable costs are tied to the amount you produce or sell. They might include raw material for a manufacturer or the cost of goods for someone in sales.
- For our simplified model we assume that variable costs are proportional to quantity. This makes our cost function linear.
- For our simplified model **variable costs = unit costs * quantity**.
- Thus **costs = fixed costs + unit costs * quantity**.



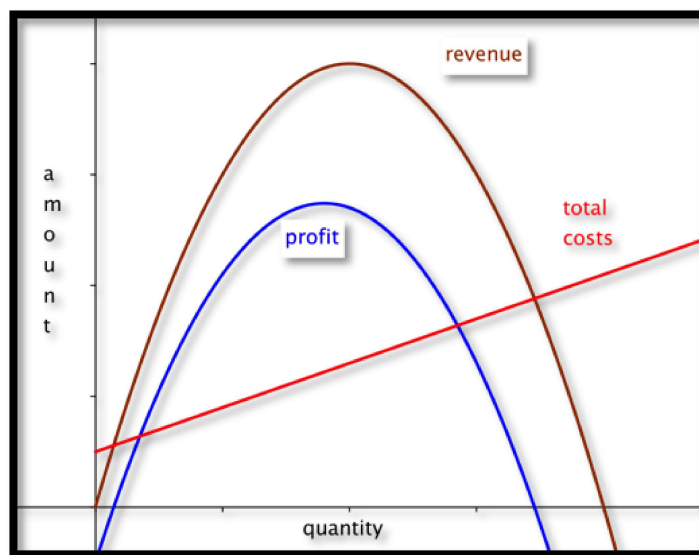
Example 2.2.2. Finding Linear Cost.

Profit:

For the third piece of the model, we look at profit. We have the simple formula

$$\text{profit} = \text{revenue} - \text{cost}.$$

For our simple examples where cost is linear and revenue is quadratic, we expect the profit function to also be quadratic, and facing down. We will obviously be interested in the spots where the profit function either crosses the axis or reaches a maximum.



Example 2.2.3. Finding Profit.

🔗 **Break-Even Point:**

- 🔗 The last example illustrates a reality of manufacturing and retail. If a business has a fixed cost or startup expense, it will have a loss if it does not sell enough.

The point at which revenues equal expenses (cost) is called the **break-even point**.

- 🔗 This is important in preparing a business proposal, because the bank will want to know if the break even point is a reasonable amount before it lends any money.

🔗 **Example 2.2.4. Find Break-Even Points.**

🔗 **Example 2.2.5. Repeat, Starting With Data.**

- 🔗 **Technical note** In business situations we often have cases where a change of quantity in the thousands only changes prices by pennies. Then our coefficients are close to zero and Excel may give formulas rounded to zero. In those cases we need to format the trendline to get more digits of accuracy.

🔗 **Example 2.2.6. Problems with using big numbers..**

🔗 **2.2.1 Exercises: Modeling Revenue, Costs, and Profit**

- 🔗 ■ For problems 1-8, given the equations of the cost and demand price function:
 - a. Identify the fixed and variable costs.
 - b. Find the revenue and profit functions.
 - c. Evaluate cost, demand price, revenue, and profit at q_0 .
 - d. Find all break-even points.
 - e. Graph the profit function over a domain that includes both break-even points. Add a textbox and label to identify the first break-even point.
- 🔗 1. Given *demand price* = $-2\text{quantity} + 20$ and *cost* = $3\text{quantity} + 10$, with $q_0 = 6$.
 - ▶ **Solution**
- 🔗 2. Given *demand price* = $-\text{quantity}/10 + 50$ and *cost* = $10\text{quantity} + 1000$, with $q_0 = 300$.

3. Given *demand price* = $-2.35\text{quantity} + 250$ and *cost* = $54.6\text{quantity} + 1234$, with $q_0 = 59$.

► Solution

4. Given *demand price* = $-0.0023\text{quantity} + 9$ and *cost* = $1.39\text{quantity} + 1398.7$, with $q_0 = 687$.

5. Given demand price and cost are the linear functions that best fit the data below and that $q_0 = 75$.

Quantity	50	100
Demand price	10	8
Cost	300	450

► Solution

6. Given demand price and cost are the linear functions that best fit the data below and that $q_0 = 110$.

Quantity	60	70	90	100
Demand Price		19		16
Cost	460		540	

7. Given demand price and cost are the linear functions that best fit the data below and that $q_0 = 75$.

Quantity	4356	4792	6503	7038
Demand price		\$1.10		\$.98
Cost	\$1190		\$1860	

► Solution

8. Given demand price and cost are the linear functions that best fit the data below and that $q_0 = 7500$.

Quantity	5378	7984	8352
Demand price	\$12.00		\$10.00
Supply price	\$31,100	\$45,100	

9. Mary has been put in charge of a school function. She estimates that there is a fixed cost of \$1000 for the site plus a cost of \$5 per person that attends. If she charges \$15 a ticket she can sell 250 tickets, but if she lowers the price to \$10 she can sell 500 tickets. Assuming the demand price is linear, what price should she charge to break even while maximizing attendance?

► Solution

■ For problems 10-12, given the cost and demand data:

- Find best fitting equations of the cost and demand curves, assuming they are both linear.
- Find the revenue and profit functions and evaluate them at the extra given value.
- Find the break-even points.

10. Given

Quantity	100	120	140	160	180	155
Cost	1015	1152	1327	1467	1651	
Demand price	21.3	18.1	14.7	12.3	8.6	

11. Given

Quantity	5021	6051	6968	7901	9023	9917	7500
Cost	80376		103874		128513	140258	
Demand price		19.69	18.78	18.05		17.61	

► Solution

12. Given

Quantity	3160	3615	4092	4462	4837	5261	5579	6000
Cost (Thousands)		90.1	126.70		197.2	234.9		
Demand price	25.31			20.91	17.04		14.37	